Dust Temperature and Spectral Index Correlation?

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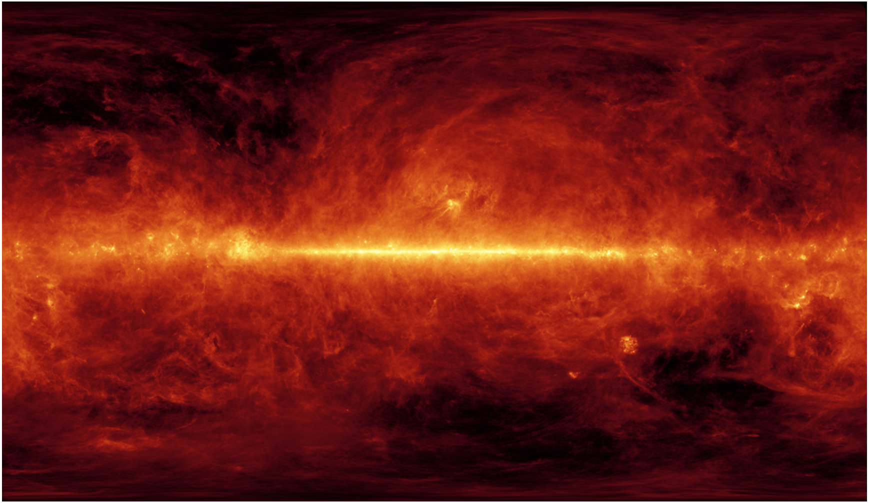
Dust Emission



Spectral Energy Distribution Fitting

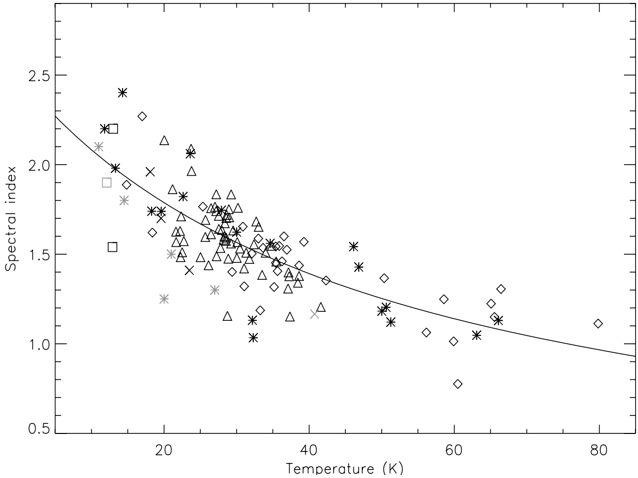
Modified Blackbody Assumption

$$S_\nu \propto C \left(\frac{\nu}{\nu_0}\right)^{\beta+3} \left(\exp\left(\frac{h\nu}{\kappa T}\right) - 1\right)^{-1}$$

Parameter in the model: (β, T).

Observations: $S_{\nu_1}, \cdots, S_{\nu_J}$.

The Empirical Inverse Correlation



A Physical Law or A Statistical One?

A Scientific Discovery

- Similar patterns on various galactic sources.
- Confirmed in different experiments by independent groups.

A Statistical Fallacy

- There are noises in the measurements.
- Estimates of parameters with noisy data are usually correlated.
- Simulation study has suggested that this might be the reason.

Two Types of Correlation

The Thought Process

$$(\beta, T) \rightarrow \text{Clean ``Data''} \rightarrow \text{Dirty Data} \rightarrow (\hat{\beta}(data), \hat{T}(data))$$

The Statistical Correlation

$$Corr(\hat{\beta}, \hat{T})$$

The Scientific Correlation

$$Corr(\beta, T)$$

Testing the Scientific Hypothesis

A Bayesian Model

$$\begin{aligned} \text{Level 1} &: \quad p(Data|\beta, T) \\ \text{Level 2} &: \quad p(\beta, T) \end{aligned}$$

Scientific Hypothesis

$$\beta \perp\!\!\!\perp T \ \text{ under } \ p(\beta, T)$$

The Statistical Model I

Likelihood

$$S_{ij} = \delta_j C_i \left(\frac{\nu_j}{\nu_0}\right)^{\beta_i+3} \left(\exp\left(\frac{h\nu_j}{\kappa T_i}\right) - 1\right)^{-1} e_{ij}$$

$$\delta_j \overset{i.i.d}{\sim} N(0, \sigma_\delta^2), C_i \overset{i.i.d}{\sim} N(\mu_c, \sigma_c^2), e_{ij} \overset{ind}{\sim} N(0, \sigma_{ij}^2).$$

Prior

$$\begin{aligned} \beta_i | T_i &\overset{i.i.d}{\sim} N(AT_i^B, \sigma_\beta^2) \\ T_i &\overset{i.i.d}{\sim} 1_{[2,150]}(T_i) dT_i \end{aligned}$$

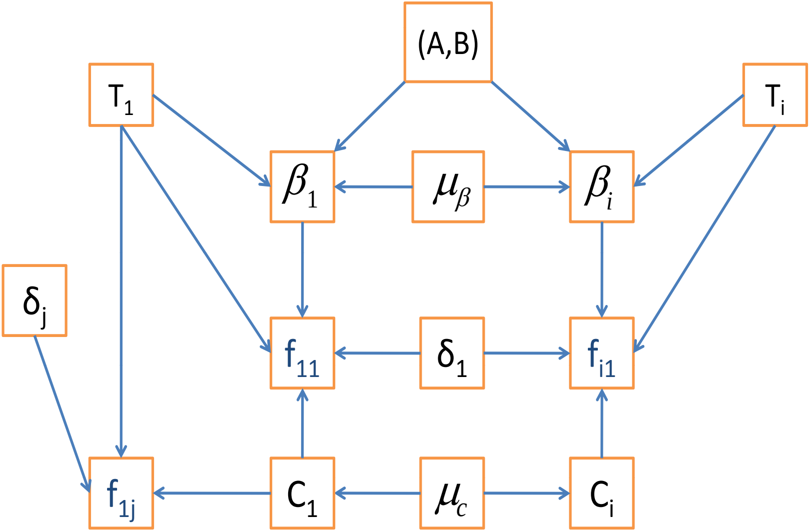
Hyper Prior

$$(\mu_c, \sigma_c^2, \sigma_\beta^2, \sigma_\delta^2) \sim d\mu_c d\ln\sigma_c^2 d\ln\sigma_b eta^2 d\ln\sigma_\delta^2$$

$$A \sim dA$$

$$B \sim 1_{[-2,2]}(B) dB$$

The Graphical Structure of the Model

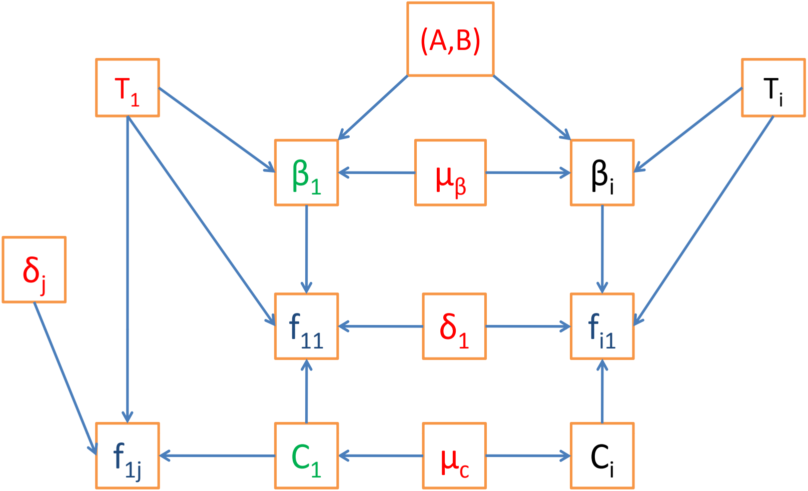


The Plain-Vanilla Gibbs Sampler

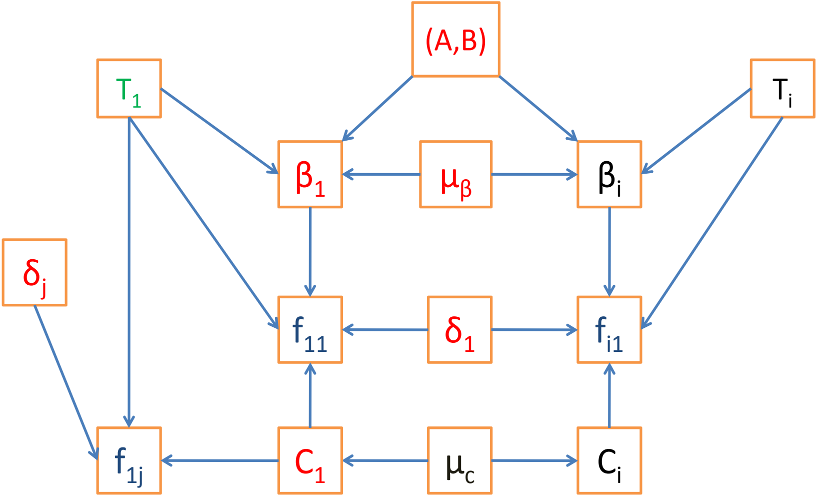
Gibbs Components

$$
\begin{aligned}
\text{Step I} &: \quad (\beta_i, C_i)|(S_{i1}, \delta_1), \cdots, (S_{iJ}, \delta_J), T_i, \mu_c, A, B \\
\text{Step II} &: \quad T_i|(S_{i1}, \delta_1), \cdots, (S_{iJ}, \delta_J), \beta_i, C_i, A, B \\
\text{Step III} &: \quad \delta_j|(S_{1j}, T_1, \beta_1, C_1), \cdots, (S_{nj}, T_n, \beta_n, C_n) \\
\text{Step IV} &: \quad \mu_c, \sigma_c^2|C_1, \cdots, C_n \\
\text{Step V} &: \quad \sigma_\delta^2|\delta_1, \cdots, \delta_J \\
\text{Step VI} &: \quad A|B, T_1, \cdots, T_n, \beta_1, \cdots, \beta_n \\
\text{Step VII} &: \quad B|A, T_1, \cdots, T_n, \beta_1, \cdots, \beta_n
\end{aligned}
$$

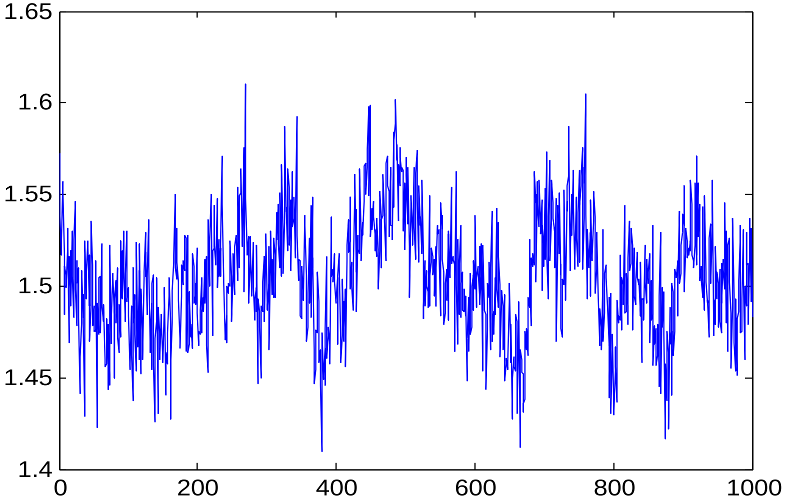
Graphical Illustration of Step I



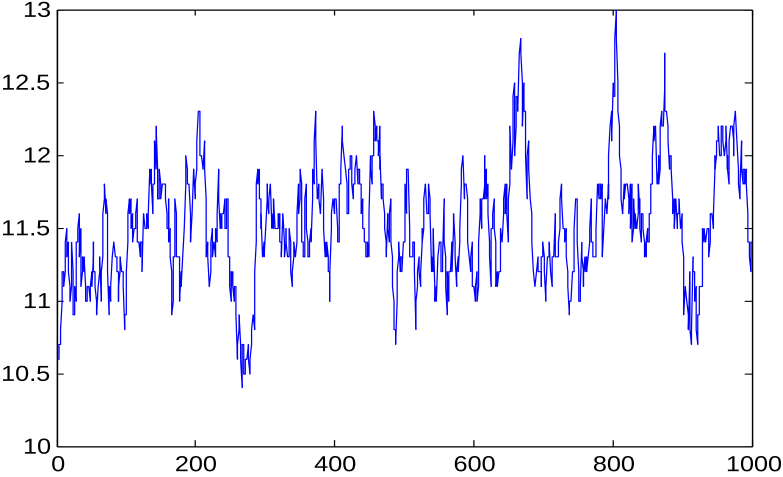
Graphical Illustration of Step II



Trace plot for β_1



Trace plot for T_1

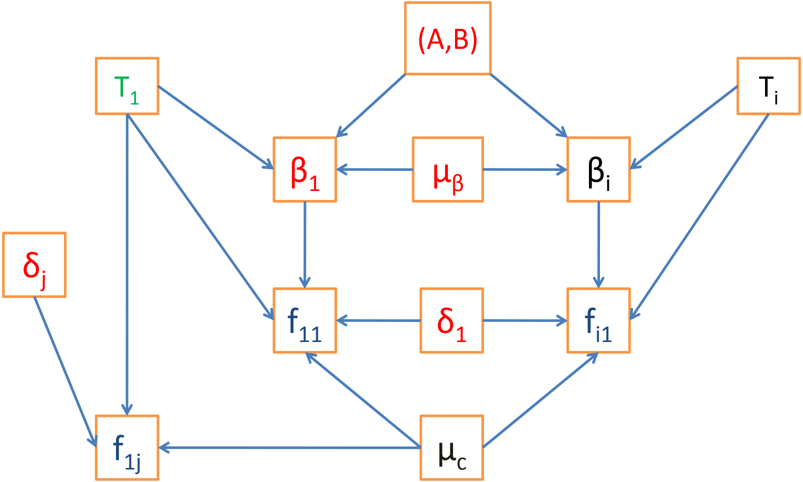


A Better Gibbs Sampler

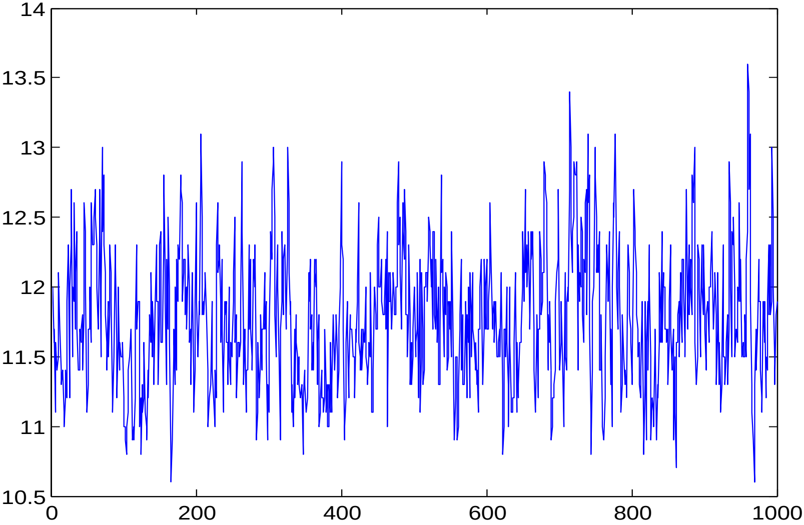
Gibbs Components

$$
\begin{aligned}
\text{Step I} &: (\beta_i, C_i)|(S_{i1}, \delta_1), \cdots, (S_{iJ}, \delta_J), T_i, \mu_c, \sigma_c^2, \sigma_\beta^2, A, B \\
\text{Step II} &: T_i|(S_{i1}, \delta_1), \cdots, (S_{iJ}, \delta_J), \beta_i, \mu_c, A, B \\
\text{Step III} &: \delta_j|(S_{1j}, T_1, \beta_1, C_1), \cdots, (S_{nj}, T_n, \beta_n, C_n) \\
\text{Step IV} &: \mu_c, \sigma_c^2|C_1, \cdots, C_n \\
\text{Step V} &: \sigma_\delta^2|\delta_1, \cdots, \delta_J \\
\text{Step VI} &: A|B, T_1, \cdots, T_n, \beta_1, \cdots, \beta_n \\
\text{Step VII} &: B|A, T_1, \cdots, T_n, \beta_1, \cdots, \beta_n
\end{aligned}
$$

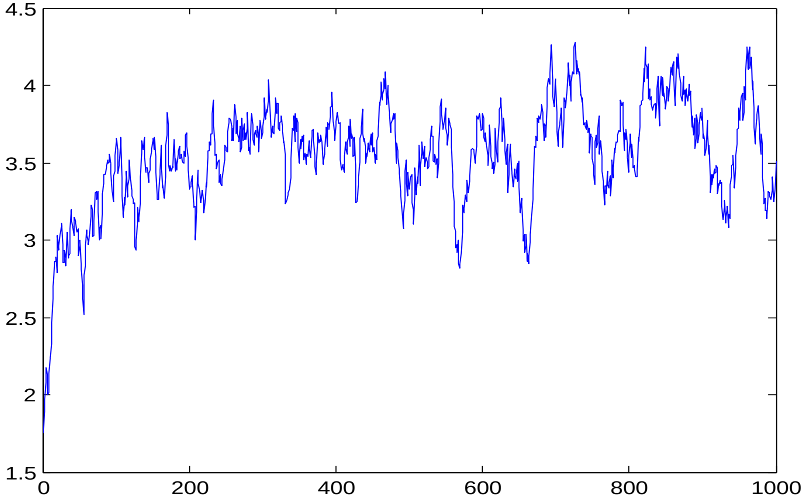
Graphical Illustration of Step II



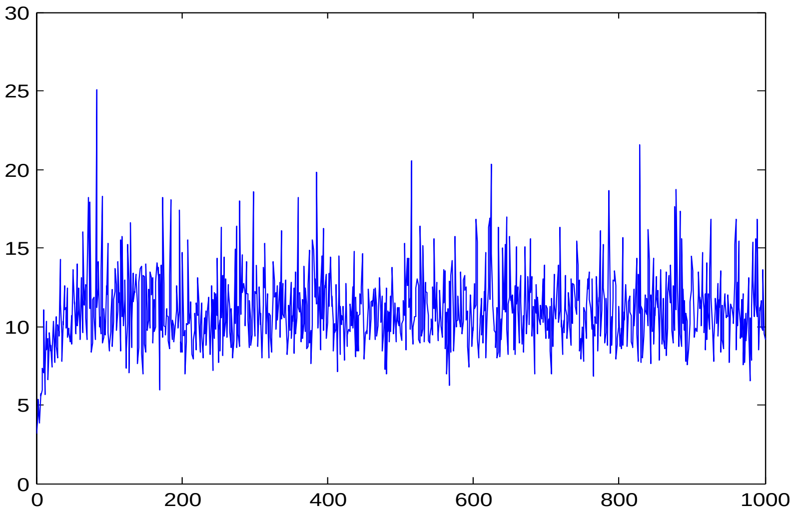
Trace plot for T_1



Application to Real Datasets



What's Wrong: Trace Plot for σ_c^2



How to incorporate the prior?

The Form of the Prior

$$\begin{aligned} (T, A, B) &\sim \pi(T, A, B) \\ \beta | T, A, B &\sim N(AT^B, \sigma_\beta^2) \end{aligned}$$

The Prior Knowledge

$$\beta \sim N(2.0, 0.2^2) \ .$$