Populations of X-ray Sources

Students: Antoine and Max
Lead Collaborators: Vinay, Andreas, Aneta and DvD

August 3, 2022

Overview (Baseline Model)

- Model a population of **X-ray sources** (e.g. from Chandra Deep Field Catalogue).
- Start by assuming all X-ray sources are located by optical survey.
- **Source intensities** estimated given photon counts from source and background region.
- **Luminosity function** specifies distribution of source intensities in a population.

- **Goal:** Estimate X-ray source intensities and obtain luminosity function.

Data

Data	Description
a_i	area of the source region
Y_i	counts collected in source region (of area a_i)
d_i	area of the background region (around source i)
X_i	background counts collected in source region (of area d_i)
e_i	telescope effective area [cm^2] at source location
r_i	proportion of photons expected to fall in source region $\equiv 1$

- Data available from the Chandra Deep Field Catalogue.
- $n = 358$ X-ray sources detected in data set.
- Observation time $\mathcal{T}$ in seconds ($\mathcal{T} = 1960631$).

Model

- In region i, observed photon counts Y_i are sum of latent background $\mathcal{B}_i$ and source $\mathcal{S}_i$

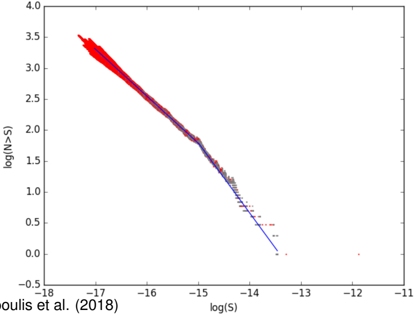
$$Y_i = \mathcal{S}_i + \mathcal{B}_i. \tag{1}$$

- Arrival of photons at detector modelled as Poisson process.
- Source model:

$$\mathcal{S}_i \big| \lambda_i \overset{\text{indep}}{\sim} \text{Poisson}(r_i e_i \lambda_i \mathcal{T}) \tag{2}$$

- **Aim:** Hierarchically estimate source intensities λ_i (count/s/cm^2) and their population.

Estimation of the Luminosity Function



Stampoulis et al. (2018)

- Piece-wise linear $\log(N) - \log(S)$ relation assumed.
- Flux S (erg/s/cm^2).

- $(\lambda_1, \ldots, \lambda_n)$ independent with double-Pareto hierarchical population **prior**

$$p(\lambda_i|\theta_1, \theta_2, \tau_1, \tau_2) = \left(\frac{\theta_1}{\tau_1}\right)\left(\frac{\lambda_i}{\tau_1}\right)^{-(\theta_1+1)} 1_{\{\tau_1 \leq \lambda_i \leq \tau_2\}} + \left(\frac{\tau_2}{\tau_1}\right)^{-\theta_1}\left(\frac{\theta_2}{\tau_2}\right)\left(\frac{\lambda_i}{\tau_2}\right)^{-(\theta_2+1)} 1_{\{\tau_2 \leq \lambda_i \leq \infty\}} \quad (3)$$

Background Model: Case 1

- Background rate ξ (count/s/pixel) **uniform** across source regions.
- Observed photon count in background modeled via

$$X\Big|\xi \sim \text{Poisson}(A\xi\mathcal{T}), \tag{4}$$

with $X = \sum_{i=1}^{n} X_i$ and $A = \sum_{i=1}^{n} d_i$ (sum of background counts, areas).

- Background count $\mathcal{B}_i$ in source region i modeled via

$$\mathcal{B}_i\Big|\xi \overset{\text{indep}}{\sim} \text{Poisson}(a_i\xi\mathcal{T}). \tag{5}$$

$\Longrightarrow$ Observed count in region i, $Y_i = \mathcal{S}_i + \mathcal{B}_i$, modeled via

$$Y_i\Big|(\lambda_i, \xi) \overset{\text{indep}}{\sim} \text{Poisson}\left((a_i\xi + r_i e_i \lambda_i)\mathcal{T}\right). \tag{6}$$

Background Model: Case 2

- Assume that background rate ξ_i varies for different source regions *i*.
- Observed photon count in the background of source *i* modeled via

$$X_i \big| \xi_i \sim \text{Poisson}(d_i \xi_i \mathcal{T}). \tag{7}$$

- Background count $\mathcal{B}_i$ in source region *i* via

$$\mathcal{B}_i \big| \xi_i \overset{\text{indep}}{\sim} \text{Poisson}(a_i \xi_i \mathcal{T}). \tag{8}$$

$\Longrightarrow$ Observed photon count in region *i*, $Y_i = \mathcal{S}_i + \mathcal{B}_i$,

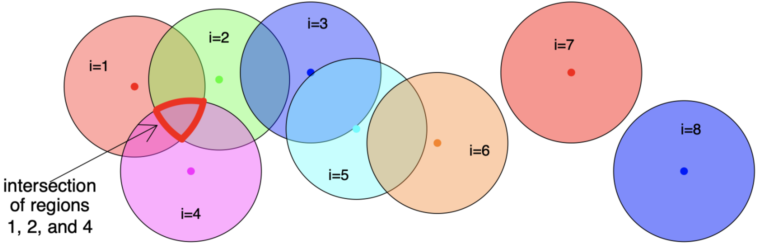
$$Y_i \big| (\lambda_i, \xi_i) \overset{\text{indep}}{\sim} \text{Poisson}\left((a_i \xi_i + r_i e_i \lambda_i)\mathcal{T}\right). \tag{9}$$

- Non-informative Gamma priors on ξ_i and $\theta_1, \theta_2, \tau_1, \tau_2$.

Extensions/Future Work

- Overlapping Sources
  - Consider segments of overlap.
- Incompleteness
  - X-ray sources existing without optical match.
  - Probability for X-ray source being included depends on its intensity (and instrumental effects).
- Unknown number of sources.
- ...

Overlapping Sources



- Photon counts $Y_{\mathcal{I}(s)}$ modelled in each of 14 segments s of overlap.
- For highlighted segment: $\mathcal{I}(s) = \{1, 2, 4\}$ with counts $Y_{\mathcal{I}(s)}$.
- $Y_{\mathcal{I}(s)}$ consists of mixture of photons from sources in s and background

$$Y_{\mathcal{I}(s)} = \sum_{i \in \mathcal{I}(s)} \mathcal{S}_{s,i} + \mathcal{B}_{\mathcal{I}(s)}. \tag{10}$$

- Then, for each segment s

$$\mathcal{S}_{s,i} \big| \lambda_i \overset{\text{indep}}{\sim} \text{Poisson}(r_{s,i} e_{s,i} \lambda_i \mathcal{T}). \tag{11}$$

Extensions/Future Work

- Overlapping Sources
  - Consider segments of overlap.
- Incompleteness
  - X-ray sources existing without optical match.
  - Probability for X-ray source being included depends on its intensity (and instrumental effects).
- Unknown number of sources.
- ...

References I

Stampoulis, V. (2018). *Bayesian estimation of luminosity distributions and model based classification of astrophysical sources*. PhD thesis, Imperial College London.

Udaltsova, I. S. (2014). *The Universe at Your Fingertips: Bayesian Modeling and Computation in Problems of Observational Cosmology*. University of California, Davis.

Wang, L., Kashyap, V. L., van Dyk, D. A., and Zezas, A. (2022). Bayesian methods for modeling source intensities. *Paper Draft*.

Thank you very much for your time!

Additional Data available:

Data	Description
a_i	area of the source region
Y_i	counts collected in source region (of area a_i)
d_i	area of the background region (around source i)
X_i	background counts collected in source region (of area d_i)
e_i	telescope effective area [cm^2]
r_i	proportion of photons expected to fall in source region $\equiv 1$
bg-sur-bri	background counts /pixel (for incompleteness correction)
off-axis	(off-axis angle - needed for the incompleteness correction)
sign	(source S/N ratio)

- Data available from the Chandra Deep Field Catalogue.
- $n = 358$ X-ray sources detected in data set.
- Observation time $\mathcal{T}$ in seconds ($\mathcal{T} = 1960631$).
- The count-rate to flux conversion for the reference point is 1.06E-11 erg/s/cm^2/cnt/s

Likelihood Functions

Background case 1:

- Likelihood function for $(\xi, \boldsymbol{\lambda})$, with $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_n)$,

$$L(\xi, \boldsymbol{\lambda} \big| \mathbf{D}) = \exp(-A\mathcal{T}\xi)\frac{(A\mathcal{T}\xi)^X}{X!}\prod_{i=1}^{n}\exp[-(a_i\xi + r_i e_i \lambda_i)\mathcal{T}]\frac{[(a_i\xi + r_i e_i \lambda_i)\mathcal{T}]^{Y_i}}{Y_i!}$$

Background case 2:

- Likelihood function for $(\boldsymbol{\xi}, \boldsymbol{\lambda})$, with $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_n)$ and $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_n)$

$$L(\boldsymbol{\xi}, \boldsymbol{\lambda} \big| \mathbf{D}) = \prod_{i=1}^{n}\exp(-a_i\mathcal{T}\xi_i)\frac{(a_i\mathcal{T}\xi_i)^{X_i}}{X_i!}\prod_{i=1}^{n}\exp[-(a_i\xi + r_i e_i \lambda_i)\mathcal{T}]\frac{[(a_i\xi + r_i e_i \lambda_i)\mathcal{T}]^{Y_i}}{Y_i!}$$