Studying Star Formation through Hierarchical Bayesian Modeling of Emission from Astronomical Dust

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Using Dust to Study Star Formation

Cold Clouds of gas and dust become unstable, collapse

Dust is important for cooling gas, aids collapse

↓

Eventually, some regions become dense and hot enough to start fusion

↓

Star is formed, disk of material forms around star

↓

Dust grains collide in disk around star, form building blocks of planets

Are the observations consistent with this picture?

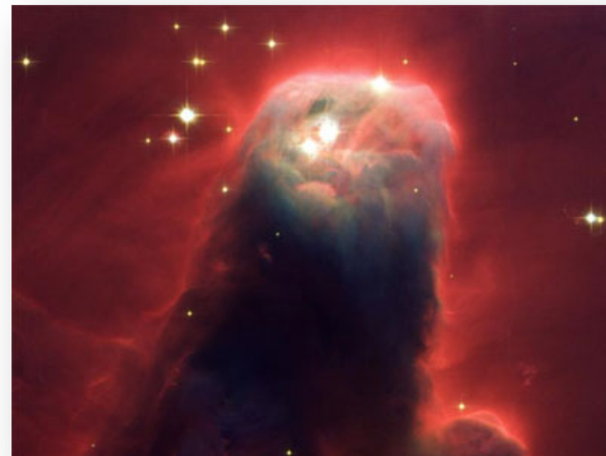
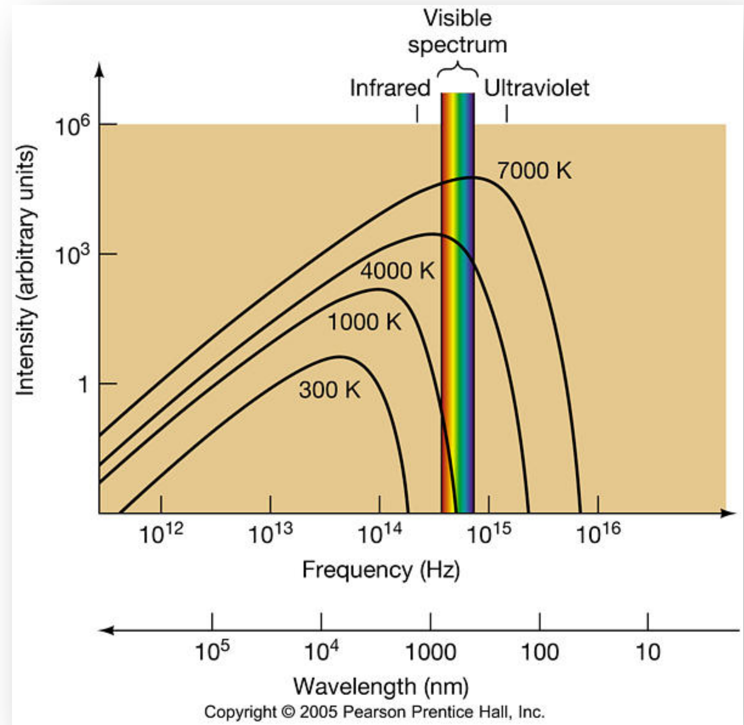


Image Credit: NASA/JPL

Thermal Emission

- Objects with a temperature T emit thermal emission
  - Idealized spectrum of thermal emission is the Planck function
  - Often called `blackbody emission'
  - Cooler object emit more energy at longer wavelengths
- Dust is an efficient radiator of thermal energy
- For dust between stars, emission is dominated by Far-IR to sub-mm wavelengths.



Mode and Normalization of BB increase as temperature increases

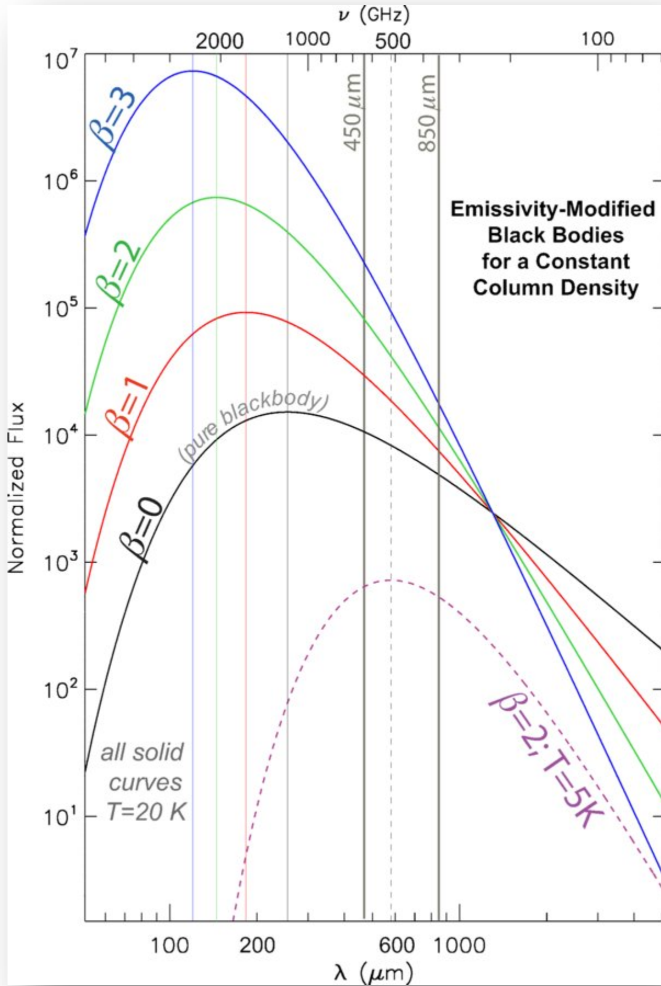
Modeling Dust Emission

- Model dust brightness as a modified black-body:

$$f(\nu) \propto N\nu^{\beta} B_{\nu}(T),\ B_{\nu}(T) \propto \nu^{3}\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]^{-1}$$

- Parameters are the dust temperature, T, the power-law modification index, β, and the column density, N

- β -> 0 as dust grains become larger

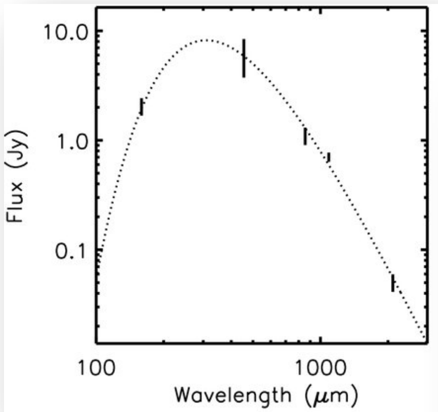


Shetty+2009

Astrophysical Expectations

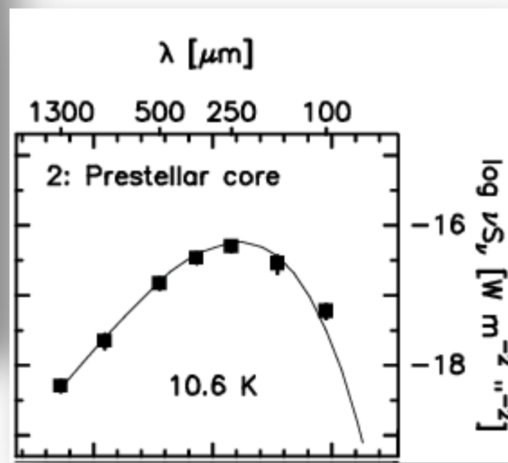
- Dust coagulation: Higher dust agglomeration in dense regions should create larger grains
  - β should decrease toward denser regions
- For dust between stars with silicate and graphite composition, $\beta \approx 2$
- For disks of dust around new stars, observations find $\beta \leq 1$.
- Temperature should decrease in higher density regions
  - Higher density regions more effectively shielded from ambient radiation field

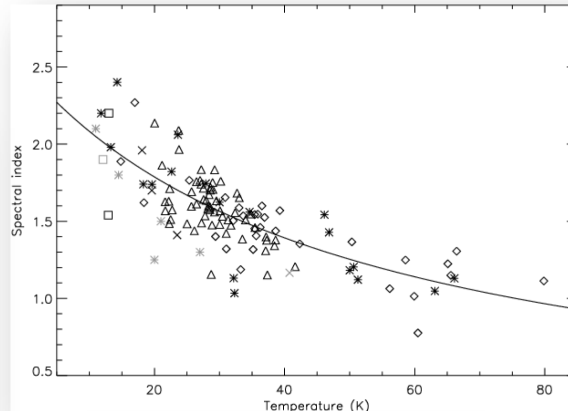
Previous Observational Results



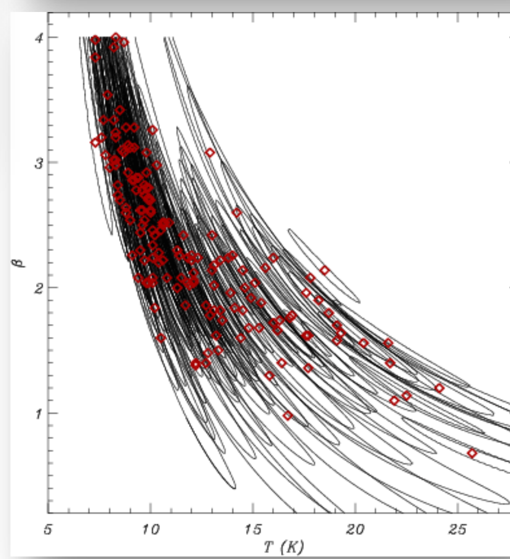
Schnee+2010

Stutz+2010





Dupac +2003



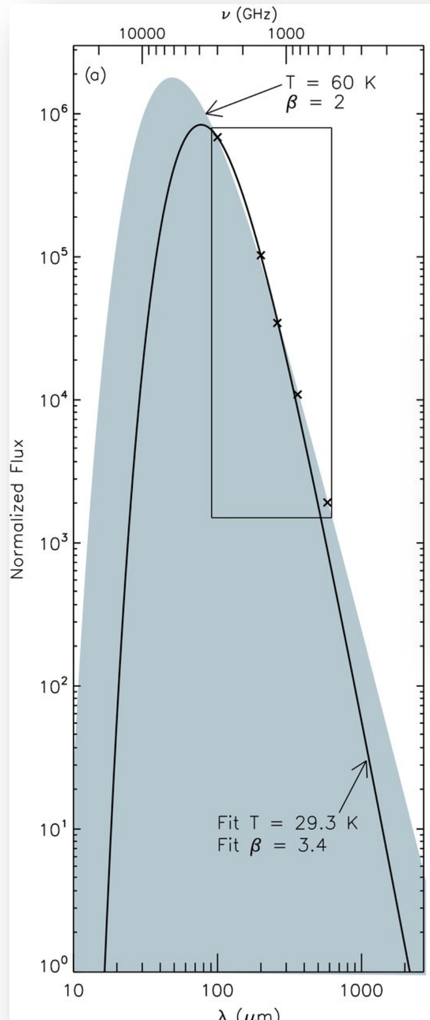
Désert et al., 2008

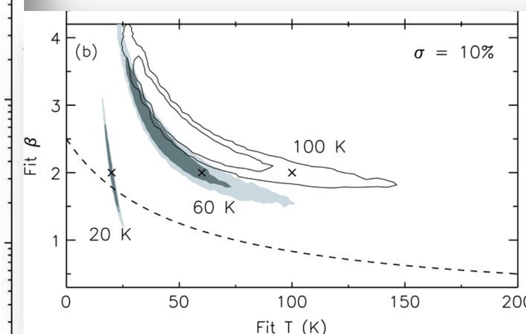
Observations find β and temperature anti-correlated (e.g., Dupac+2003, Desert+2008, Anderson+2010, Paradis+2010,Planck Collaboration 2011)

Anti-Correlation is Unexpected

Unexpected: Why a β–T anti-correlation?

- β and T estimated by minimizing weighted squared error (i.e., χ^2)
- Typically only 5–10 data points for estimating 3 parameters
- Errors on estimated β and T large, highly anti-correlated
- Errors bias inferred correlation, may lead to spurious anti-correlation (Shetty+2009)





Shetty+2009

The Measurement Model

$$y_{ij} = \delta_j S_j(N_i, T_i, \beta_i) + \varepsilon_{ij}$$ ← Measured fluxes

$$S_j(N_i, T_i, \beta_i) \propto N_i \left(\frac{\nu_j}{\nu_0}\right)^{\beta_i} B_{\nu_j}(T_i)$$ ← Astrophysical model for flux

$$\varepsilon_{ij} \sim N(0, \sigma_{ij}^2), \quad \log \delta_j \sim N(0, \tau_j^2)$$ ← Measurement errors

y_{ij} : Measured flux value at j^{th} wavelength for i^{th} data point
$i = 1,\ldots,n \quad j = 1,\ldots,p$
N_i, T_i, β_i : model parameters for i^{th} data point
ν_j : Frequency corresponding to j^{th} observational wavelength
δ_j = Calibration error for j^{th} observational wavelength
ϵ_{ij} = Measurement noise for y_{ij}

Our Hierarchical Model

Joint distribution of log N, log T, and β modeled as a multivariate Normal distribution

$$\mu,\Sigma \sim Uniform$$

$$\log N_i, \log T_i, \beta_i \mid \mu,\Sigma \sim N(\mu,\Sigma)$$

$$\log \delta_j \sim N(0,\tau_j^2)$$

$$y_{ij} \mid N_i, T_i, \beta_i, \delta_j \sim N(\delta_j S_j(N_i, T_i, \beta_i), \sigma_{ij}^2)$$

τ_j and σ_{ij} are considered known and fixed

Calibration Errors are nuisance parameters

Naïve MCMC Sampler

1. Update calibration error, do Metropolis-Hastings (M-H) update with proposal

$$\log \delta_j^{\text{prop}} \sim N(\log \bar{\delta}_j, Var(\bar{\delta}_j)/\bar{\delta}_j^2)$$

$$\bar{\delta}_j = Var(\bar{\delta}_j)\left(\frac{\bar{\delta}_j^{\text{data}}}{v_j^{\text{data}}} + \frac{1}{\tau_j^2}\right), \quad Var(\bar{\delta}_j) = \left(\frac{1}{v_j^{\text{data}}} + \frac{1}{\tau_j^2}\right)^{-1}$$

$$\bar{\delta}_j^{\text{data}} = v_j^{\text{data}} \sum_{i=1}^{n} \frac{y_{ij}}{S_j(N_i,T_i,\beta_i)}\left(\frac{\sigma_{ij}}{S_j(N_i,T_i,\beta_i)}\right)^{-2}, \quad v_j^{\text{data}} = \left(\sum_{i=1}^{n}\left(\frac{\sigma_{ij}}{S_j(N_i,T_i,\beta_i)}\right)^{-2}\right)^{-1}$$

2. Update values of N_i, T_i, and β_i using M-H update with multivariate normal proposal density
3. Do Gibbs update of μ and Σ using standard updates for mean and covariance matrix of Normal distribution

Ancillary-Sufficiency Interweaving Strategy

- Naïve MCMC sampler is very slow due to strong dependence between δ, log N, β, and μ:

$$E(y_{ij} \mid N_i, T_i, \beta_i, \delta_j) \propto \delta_j N_i \left(\frac{\nu_j}{\nu_0} \right)^{\beta_i} B_{\nu_j}(T_i)$$

- Use Ancillary-Sufficiency Interweaving Strategy (ASIS, Yu & Meng 2011) to break dependence

References:

Yu, Y, & Meng, X-L, 2011, JCGS (with discussion), 20, 531

Kelly, B.C., 2011, JCGS, 20, 584

Introduce Sufficient Augmentation

- Original data augmentation is an *ancillary* augmentation for calibration error: δ and (β,N,T) independent in their prior

- Introduce a *sufficient* augmentation for calibration error:

$$\log\tilde{\delta}_i = \log\delta + \mathbf{X}\theta_i$$

$$\mathbf{X} = \begin{pmatrix} 1 & \log\frac{\nu_1}{\nu_0} \\ \vdots & \vdots \\ 1 & \log\frac{\nu_p}{\nu_0} \end{pmatrix}, \quad \theta_i = (\log N_i, \beta_i)^T$$

Sufficient Augmentation Model

- New Model is

$$
\begin{aligned}
&y_{ij} \mid T_i, \tilde{\delta}_{ij} \sim N(\tilde{\delta}_{ij} B_j(T_i), \sigma_{ij}^2) \\
&\log \tilde{\delta}_i \mid \log \tilde{\delta}_k, \theta_i, \theta_k = \log \tilde{\delta}_k - \mathbf{X}(\theta_k - \theta_i), i \neq k \\
&\log \tilde{\delta}_k \mid \theta_k \sim N(\mathbf{X}\theta_{\mathbf{k}}, V_\delta), \quad V_\delta = \mathrm{diag}(\boldsymbol{\tau}_{\mathrm{j}}^2) \\
&(\theta_i, \log T_i) \mid \mu, \Sigma \sim N(\mu, \Sigma)
\end{aligned}
$$

ASIS for this problem

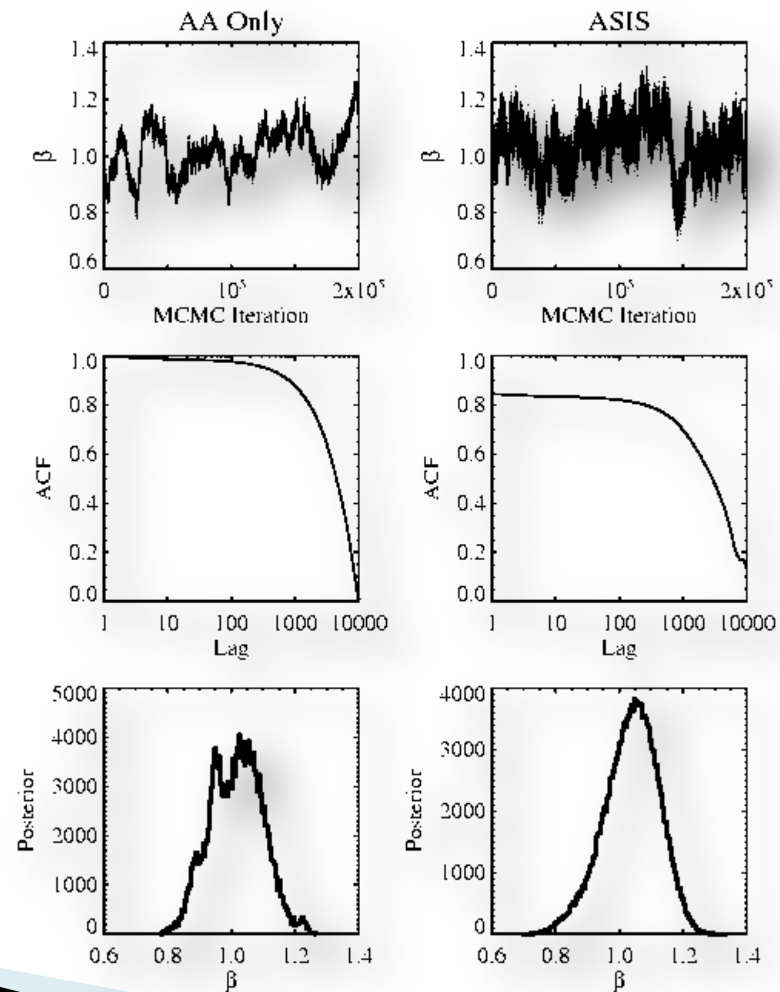
1. Update calibration error as before
2. Update δ, log N, and β under SA:

$$
\begin{aligned}
&\theta_k^{new} \mid \tilde{\delta}_k \sim N(\hat{\theta}_k, V_k) \\
&\hat{\theta}_k = V_k \mathbf{X}^T V_\delta^{-1} \tilde{\delta}_k \\
&V_k = (\mathbf{X}^T V_\delta^{-1} \mathbf{X})^{-1} \\
&\theta_i^{new} \mid \theta_k^{new}, \tilde{\delta}_k, \tilde{\delta}_i = \theta_k^{new} + (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T (\tilde{\delta}_i - \tilde{\delta}_k) \\
&\delta = \tilde{\delta}_k - \mathbf{X} \theta_k^{new}
\end{aligned}
$$

(also need to update μ appropriately)

3. Update log N, log T, and β as before under AA
4. Update μ and Σ

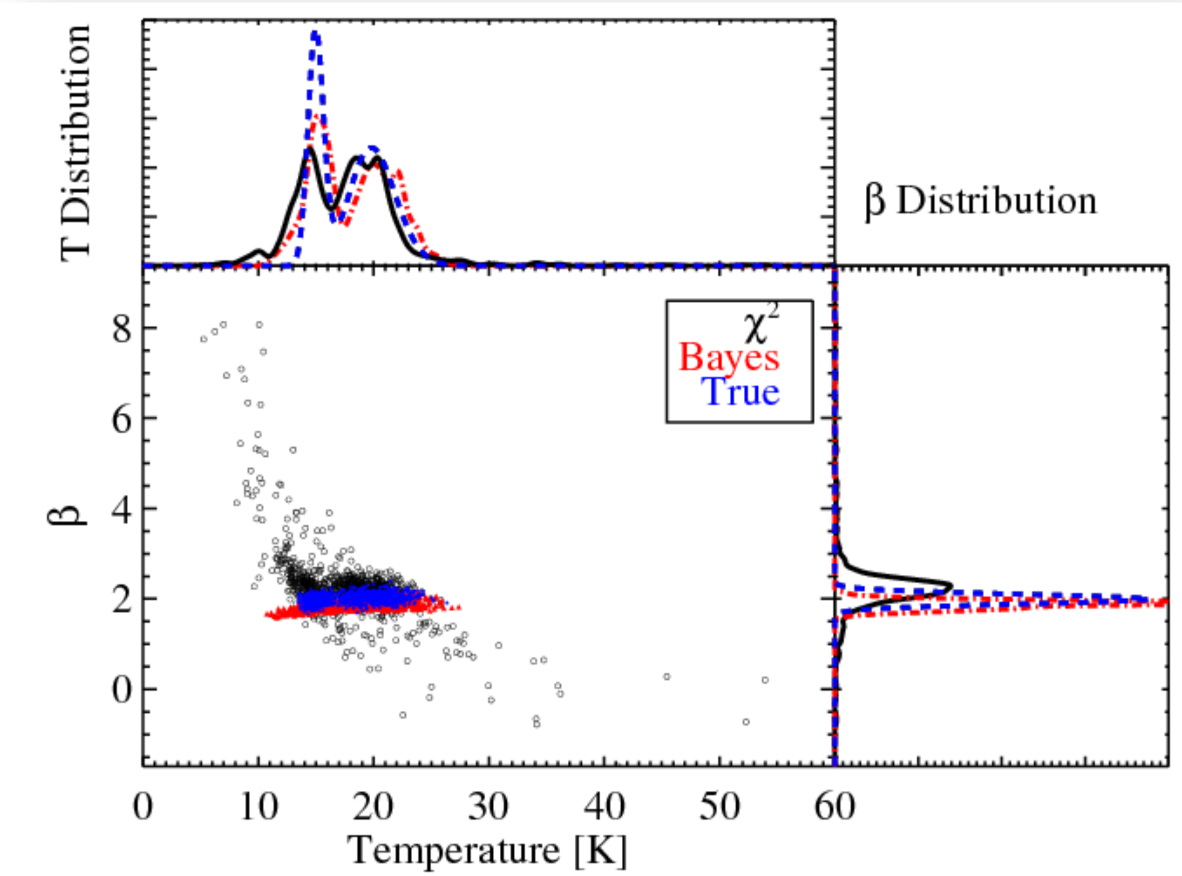
ASIS Significantly Improves MCMC Efficiency for this Problem



MCMC results for data point with highest S/N, 200,000 iterations

Kelly 2011

Test: Comparison of Bayesian and χ^2 methods for Intrinsic Correlation

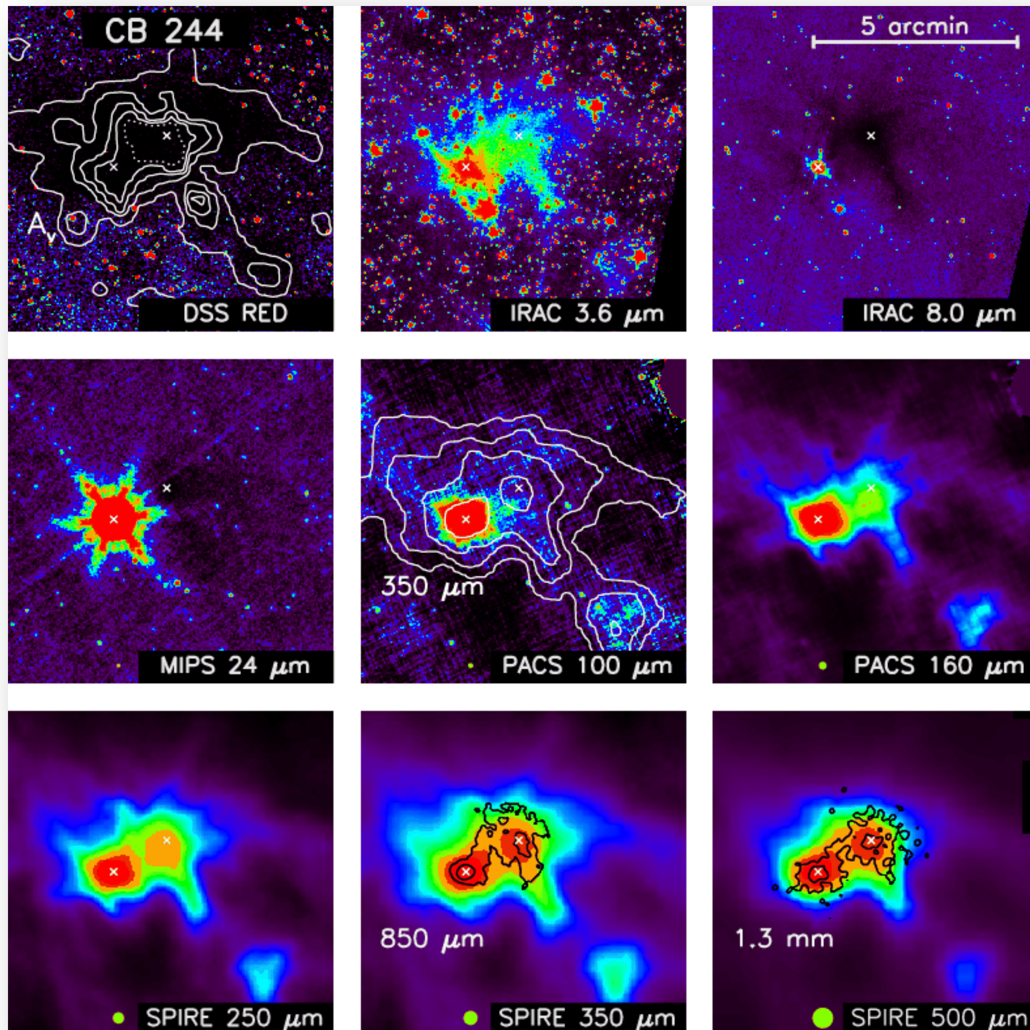


Rank correlation coefficients:

True = 0.33
χ^2 = −0.45±0.03
Bayes = 0.23±0.08

Application: Star-Forming Bok Globule CB244

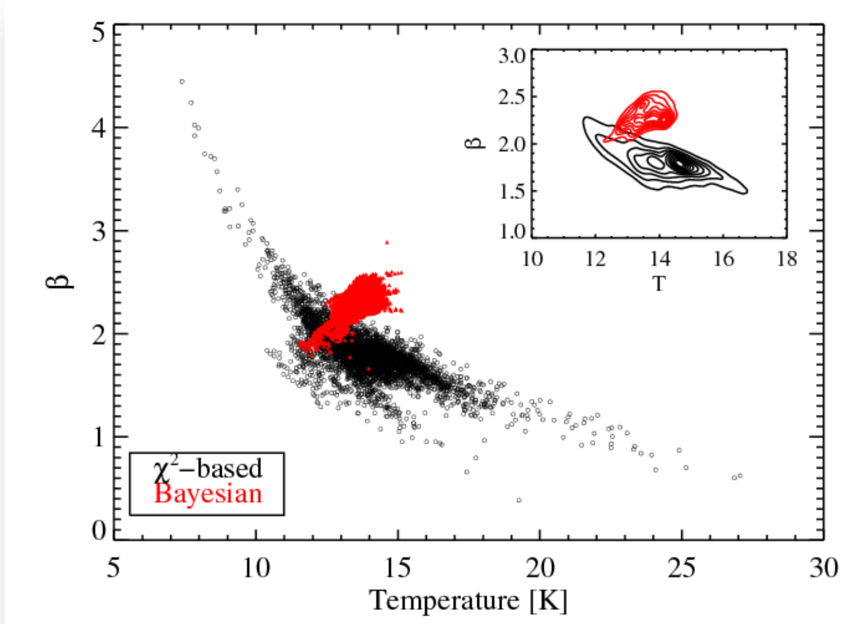
CB244: Small, nearby molecular cloud containing a low-mass protostar and starless core



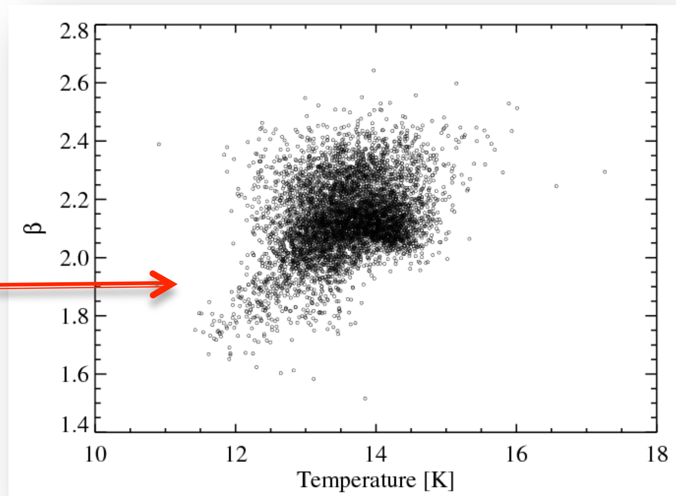
Stutz+2010

CB244 Results:

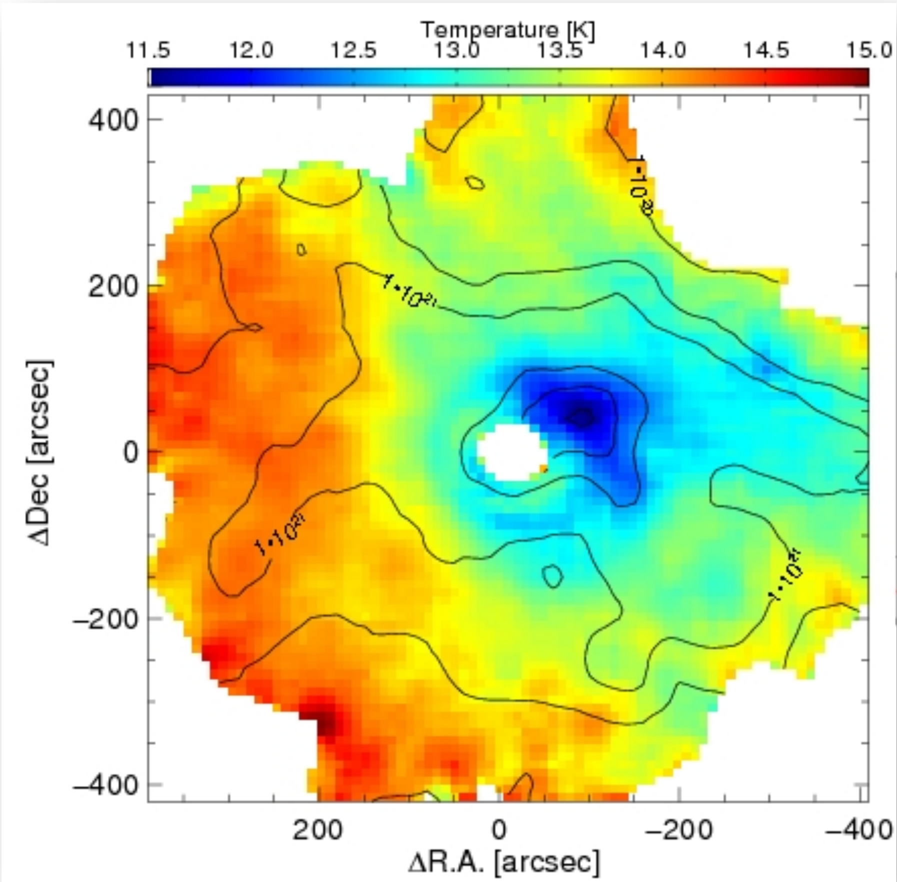
- β and T ***anti-correlated*** for χ^2-based estimates, caused by noise
- β and T ***correlated*** for Bayesian estimates, opposite what has been seen in previous work

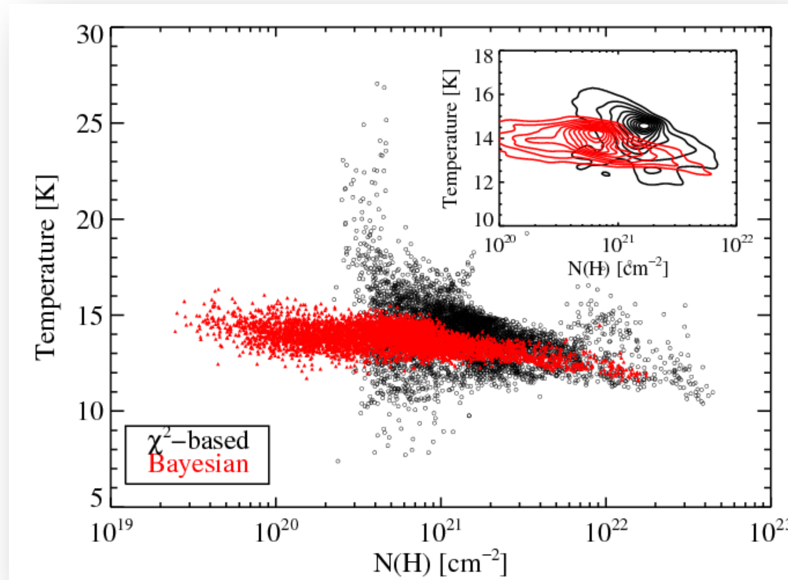


Random Draw from Posterior Probability Distribution



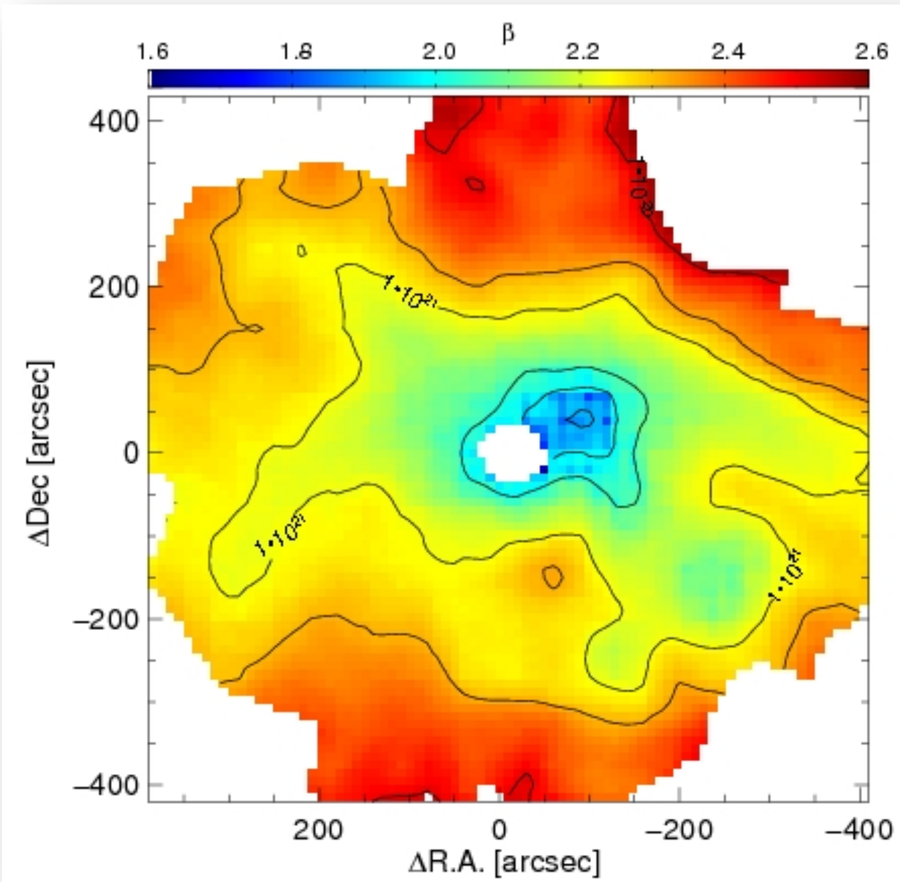
CB244: Temperature and Column Density Maps

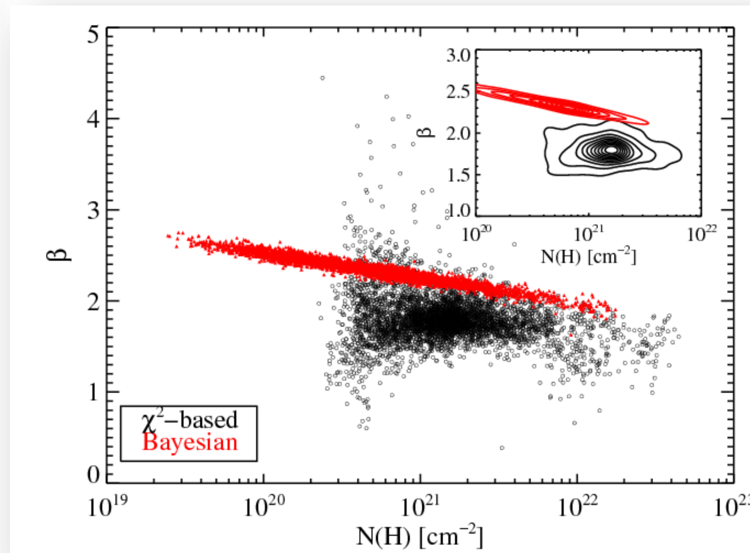




Temperature tends to decrease toward central, denser regions

CB244: Column Density and β Maps





β decreases toward central denser regions

Evidence for Grain Growth?

- β is expected to be smaller for larger grains
- $\beta < 1$ for protoplanetary disks, denser than starless cores
- For CB244 we find β is smaller in denser regions
- Suggests dust grains begin growing on large scales before star forms
- Qualitatively consistent with spectroscopic features seen in mid-IR observations of starless cores (Stutz+2009)

Summary

- Introduced a Bayesian method that correctly recovers intrinsic correlations involving N,β, and T, in contrast to traditional χ^2
- Developed an ancillarity-sufficiency interweaving strategy to make problem computationally tractable
- For CB244, our Bayesian method finds that β and T are correlated, opposite that of χ^2
- For CB244, we also find that β increases toward the central dense regions, while temperature decreases
- Increase in β with column density may be due to growth of dust grains through coagulation
- Bayesian method led to very different scientific conclusions compared to naïve χ^2 method, illustrating importance of proper statistical modeling for complex astronomical data sets