Combining Computer Models to Account for Mass Loss in Stellar Evolution

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Statistics 310

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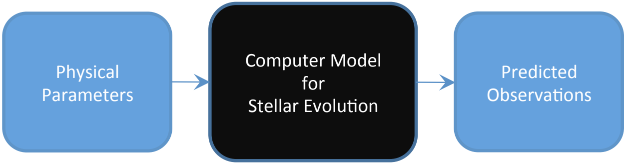
Statistical Analysis of Stellar Evolution

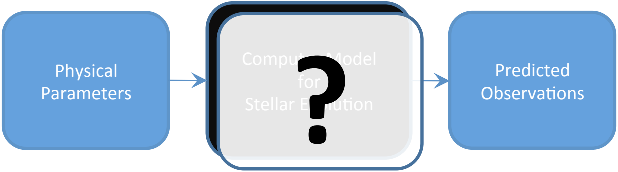
- Statistical analysis of stellar evolution relies on complicated models of the physical aging processes of stars
- Like a sampling distribution, these models predict observed quantities as a function of unknown parameters



- These models vary in complexity: some are solutions of coupled partial differential equations, some have simple analytic expressions
- We use tabulated versions of the more complex models, evaluated over a grid of parameter values

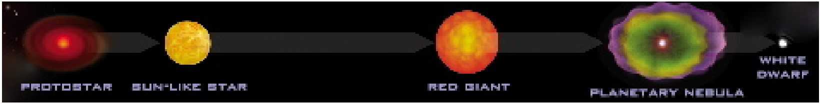
Opening the Black Box

- Typically treat computer models as deterministic black box models
- We want to open the black box and see what the data can tell us about internal model components



- **What can we learn about the processes of stellar evolution from observations of star clusters?**
- We focus on the mass loss that stars experience along the way to their final stage as white dwarfs

Evolution of a Sun-like Star



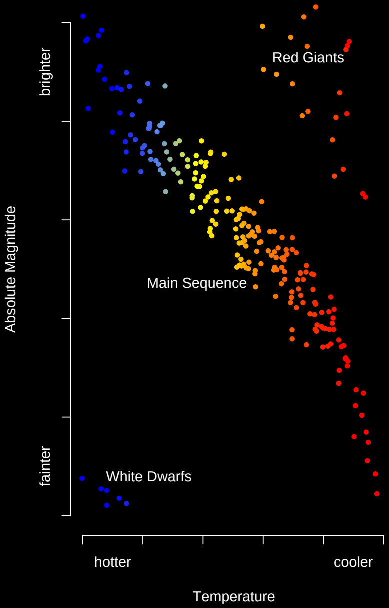
1. **Main sequence**
   - powered by hydrogen fusion
2. **Red giant**
   - no more hydrogen fuel, so the star cools and swells and sheds its outer layers
3. **Planetary nebula**
   - remaining hot core ionizes the outer layers that have been ejected
4. **White dwarf**
   - once the outer layers are gone, the hot, dense core remains

Initial-Final Mass Relation (IFMR)

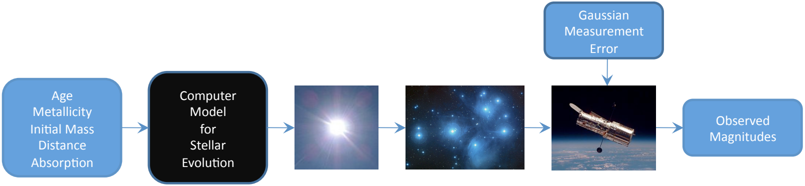
- White dwarf mass $<$ progenitor star mass
- The mapping between the progenitor mass and the white dwarf mass is called the **initial-final mass relation** (IFMR)
- Key ingredient in physics-based models of stellar evolution
- Interesting complication: relationship between two unobserved quantities (only one even observable)

Data

- Observe stars through different photometric filters
- Focus on clusters of stars with the same age, chemical composition (metallicity), distance, absorption
- Stars have different initial masses
- Initial masses govern their rates of evolution $\rightarrow$ see a snapshot of stars in different stages of evolution


brighter
fainter
Absolute Magnitude
Red Giants
Main Sequence
White Dwarfs
hotter
cooler
Temperature

Basic Likelihood



- $\mathbf{Y}_i$ = vector of observed magnitudes through different filters
- M_i = the mass of star i
- $\boldsymbol{\theta}$ = vector of cluster parameters
- $\mathbf{G}_{\mathrm{MS/RG}}(M_i, \boldsymbol{\theta})$ = the stellar evolution model for main sequence stars
- Observational uncertainties $\boldsymbol{\Sigma}_i$ assumed known
- Gaussian errors:

$$\mathbf{Y}_i | M_i, \boldsymbol{\theta}, \boldsymbol{\alpha}, \boldsymbol{\Sigma}_i \overset{indep}{\sim} N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i),$$

$$\boldsymbol{\mu}_i = \mathbf{G}_{\mathrm{MS/RG}}(M_i, \boldsymbol{\theta}) \text{ if star } i \text{ is a main sequence star}$$

Basic Likelihood

- $f(M_i, \boldsymbol{\alpha})$ = the initial-final mass relation
- $\boldsymbol{\alpha}$ = vector of IFMR parameters
- $\mathbf{G}_{\mathrm{WD}}(M_i, \boldsymbol{\theta}, \boldsymbol{\alpha})$ = the stellar evolution model for white dwarfs
- Gaussian errors:

$$\mathbf{Y}_i | M_i, \boldsymbol{\theta}, \boldsymbol{\alpha}, \boldsymbol{\Sigma}_i \overset{indep}{\sim} N(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i),$$

$$\boldsymbol{\mu}_i = \begin{cases} \mathbf{G}_{\mathrm{MS/RG}}(M_i, \boldsymbol{\theta}) & \text{if star } i \text{ is a main sequence star} \\ \mathbf{G}_{\mathrm{WD}}(M_i, \boldsymbol{\theta}, \boldsymbol{\alpha}) & \text{if star } i \text{ is a white dwarf} \end{cases}$$

Binaries and Field Stars

Binary Systems

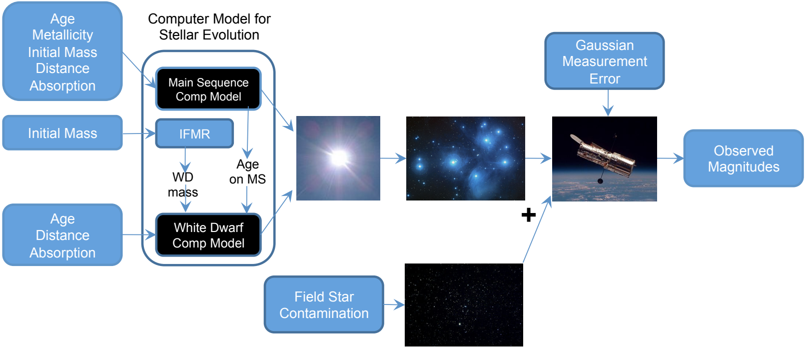
- Between 1/3 and 1/2 of stars are binary systems that appear as one star
- Luminosities of component stars sum
- Magnitude $= -2.5 \log_{10}(\text{luminosity})$
- For main sequence-main sequence binaries,

$$\mu_{ij} = -2.5 \log_{10}\left(10^{-G_{\text{MS/RG},j}(M_{i1},\boldsymbol{\theta})/2.5} + 10^{-G_{\text{MS/RG},j}(M_{i2},\boldsymbol{\theta})/2.5}\right)$$

- All main sequence stars are modeled as binaries

Field Stars

- Appear in observational field of view, but not part of cluster
- Mixture model, with field stars assumed uniformly distributed in magnitude space


Age
Metallicity
Initial Mass
Distance
Absorption
Initial Mass
Age
Distance
Absorption
Computer Model for
Stellar Evolution
Main Sequence
Comp Model
IFMR
WD
mass
Age
on MS
White Dwarf
Comp Model
Field Star
Contamination
+
Gaussian
Measurement
Error
Observed
Magnitudes

Component Computer Models

- If star is a white dwarf, the **MS/RG computer model** returns how long it lived as a main sequence and red giant star (the progenitor age).

$$\phi_{\text{prog age}} = \mathbf{F}_{\mathrm{MS/RG}}(\theta_{[\text{Fe/H}]}, M)$$

- The **white dwarf cooling model** computes the effective temperature and radius of the star as a function of its cooling age (total age minus progenitor age) and its current mass.

$$(\phi_{T_{\text{eff}}}, \phi_{\text{radius}}) = \mathbf{F}_{\mathrm{cooling}}(\theta_{\text{age}} - \phi_{\text{prog age}}, M_{\text{WD}})$$

Component Computer Models

- The log of the gravitational force experienced at the surface of the white dwarf is computed using **Newton's law**:

$$\phi_{\log g} = \log_{10}(G\, M_{\mathrm{WD}}/{\phi_{\mathrm{radius}}}^2)$$

- The **white dwarf atmosphere model** uses the surface gravity and the effective temperature to derive the emergent spectrum of the star's atmosphere as a function of wavelength. The model then integrates the emergent spectrum over the filter response to calculate the modeled magnitudes.

$$\boldsymbol{\mu} = \mathbf{F}_{\text{atmosphere}}(\phi_{T_{\mathrm{eff}}}, \phi_{\log g})$$

Component Computer Models

$$
\begin{aligned}
\phi_{\text{prog age}} &= \mathbf{F}_{\mathrm{MS/RG}}(\theta_{[\text{Fe/H}]}, M) \\
(\phi_{T_{\text{eff}}}, \phi_{\text{radius}}) &= \mathbf{F}_{\mathrm{cooling}}(\theta_{\text{age}} - \phi_{\text{prog age}}, M_{\text{WD}}) \\
\phi_{\log g} &= \log_{10}(G\, M_{\text{WD}} / {\phi_{\text{radius}}}^2) \\
\boldsymbol{\mu} &= \mathbf{F}_{\mathrm{atmosphere}}(\phi_{T_{\text{eff}}}, \phi_{\log g})
\end{aligned}
$$

Component Computer Models

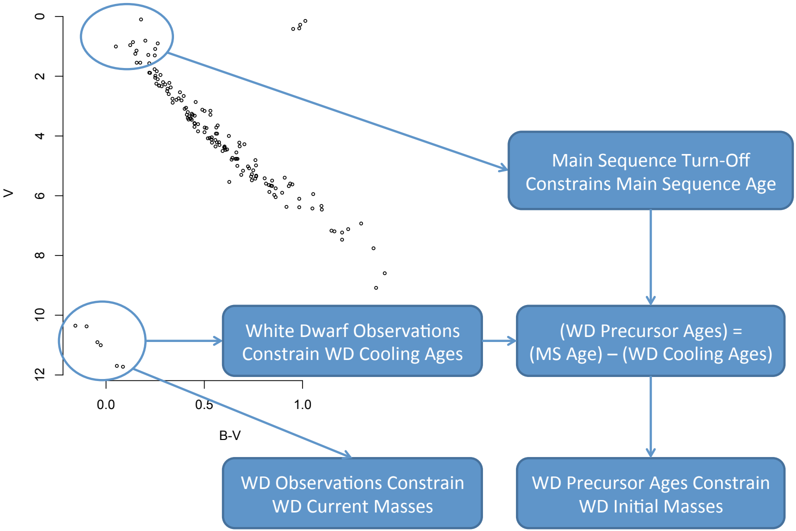
$$
\begin{aligned}
\phi_{\text{prog age}} &= \mathbf{F}_{\mathrm{MS/RG}}(\theta_{\text{[Fe/H]}}, M) \\
(\phi_{T_{\text{eff}}}, \phi_{\text{radius}}) &= \mathbf{F}_{\mathrm{cooling}}(\theta_{\text{age}} - \phi_{\text{prog age}}, M_{\text{WD}}) \\
\phi_{\log g} &= \log_{10}(G\, M_{\text{WD}} / \phi_{\text{radius}}{}^2) \\
\boldsymbol{\mu} &= \mathbf{F}_{\mathrm{atmosphere}}(\phi_{T_{\text{eff}}}, \phi_{\log g})
\end{aligned}
$$

Component Computer Models

$$\begin{aligned}
\phi_{\text{prog age}} &= \mathbf{F}_{\mathrm{MS/RG}}(\theta_{[\mathrm{Fe/H}]}, M) \\
(\phi_{T_{\mathrm{eff}}}, \phi_{\text{radius}}) &= \mathbf{F}_{\mathrm{cooling}}(\theta_{\text{age}} - \phi_{\text{prog age}}, M_{\mathrm{WD}}) \\
\phi_{\log g} &= \log_{10}(G\, M_{\mathrm{WD}} / \phi_{\text{radius}}{}^2) \\
\boldsymbol{\mu} &= \mathbf{F}_{\mathrm{atmosphere}}(\phi_{T_{\mathrm{eff}}}, \phi_{\log g})
\end{aligned}$$

Parameterizing the IFMR

- We let the IFMR be a deterministic function of the initial mass M and parameters $\boldsymbol{\alpha}$:
$$M_{\mathrm{WD}} = f(M, \boldsymbol{\alpha})$$
- We primarily consider a linear IFMR
- Simple functional forms are reasonable because visible white dwarfs in any particular cluster will typically span a relatively narrow range of initial masses


Main Sequence Turn-Off Constrains Main Sequence Age
White Dwarf Observations Constrain WD Cooling Ages
(WD Precursor Ages) = (MS Age) – (WD Cooling Ages)
WD Observations Constrain WD Current Masses
WD Precursor Ages Constrain WD Initial Masses
V
B-V
0
2
4
6
8
10
12
0.0
0.5
1.0

Prior Distributions

- Primary mass:

$$\log_{10}(\text{mass}) \sim N(-1.02, 0.677^2)\ , \qquad 0.1M_\odot < \text{mass} < 8.0M_\odot$$

  based on population distribution
- Uniform on the ratio of smaller to larger mass
- Uniform on $\log_{10}(\text{age})$ between limits of stellar evolution models
- Cluster membership prior probabilities come from external information (when available)
- Informative priors on metallicity, distance, and absorption
- Prior distribution on the IFMR parameters α is uniform on the region corresponding to monotonically increasing IFMRs

Statistical Computation

- At least $3N + 3$ parameters for cluster with N stars
- Local modes based on choices of cluster members vs field stars
- Joint posterior

$$p(\boldsymbol{\theta}, \boldsymbol{\alpha}, \mathbf{M}, \mathbf{R}, \mathbf{Z} \mid \mathbf{Y}) \propto p(\boldsymbol{\theta}, \boldsymbol{\alpha}) \prod_{i=1}^{N} \Big\{ [\pi_i p_c(\mathbf{Y}_i \mid M_i, R_i, \boldsymbol{\theta}, \boldsymbol{\alpha}) p_c(M_i, R_i)]^{Z_i} \times [(1 - \pi_i) p_f(\mathbf{Y}_i) p_f(M_i, R_i)]^{1-Z_i} \Big\}$$

  - $\mathbf{Z}$ = vector of cluster membership indicators
  - π_i = prior probability of cluster membership for star i
  - $\mathbf{R}$ = vector of ratios of secondary to primary mass
  - p_c = cluster star likelihood or prior
  - p_f = field star likelihood or prior

Statistical Computation

- Marginal posterior

$$p(\boldsymbol{\theta}, \boldsymbol{\alpha} \mid \mathbf{Y}) = \int \cdots \int \left(\sum_{Z_1} \cdots \sum_{Z_N} p(\boldsymbol{\theta}, \boldsymbol{\alpha}, \mathbf{M}, \mathbf{R}, \mathbf{Z} \mid \mathbf{Y}) \right) d\mathbf{M} d\mathbf{R}$$

$$\propto p(\boldsymbol{\theta}, \boldsymbol{\alpha}) \prod_{i=1}^{N} \Big\{ \pi_i \int \int p_c(\mathbf{Y}_i \mid M_i, R_i, \boldsymbol{\theta}, \boldsymbol{\alpha}) p_c(M_i, R_i) dM_i dR_i + (1 - \pi_i) p_f(\mathbf{Y}_i) \Big\}$$

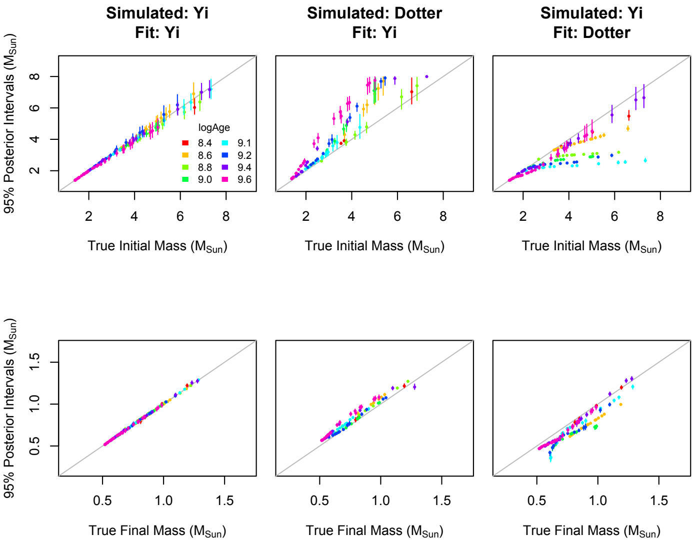
- Because of conditional independence, marginalizing out nuisance parameters involves lots of 1- and 2-dimensional integrals over compact regions (e.g. $M_i \in [0.15, 8.0]$, $R_i \in [0, 1]$), which can be numerically approximated in parallel
- Use MCMC on the lower dimensional $(\boldsymbol{\theta}, \boldsymbol{\alpha})$

Sensitivity to Misspecification

- Deterministic models $\mathbf{G}_{\text{MS/RG}}$ are assumed known, but there are uncertainties, different implementations, etc.
- Performed a simulation to test the sensitivity of inferences to misspecification of $\mathbf{G}_{\text{MS/RG}}$
- Used the models of Yi et al. (2001) and Dotter et al. (2008)
- Simulated eight clusters under both sets of models at different ages using the linear IFMR of Williams et al. (2009):

$$M_{\text{WD}} = 0.339 + 0.129M$$

- Fit the clusters using both sets of models and a linear IFMR

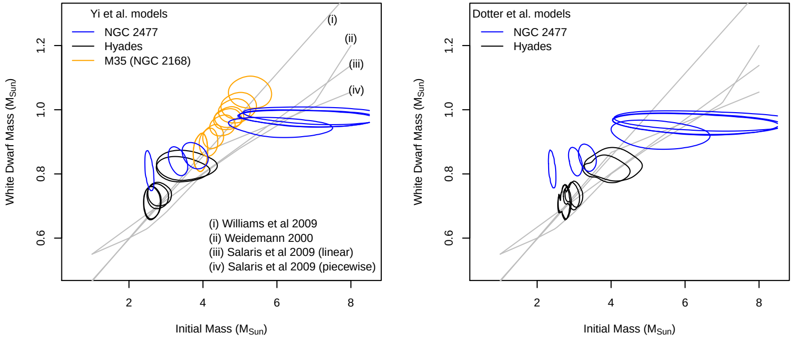


Simulated: Yi
Fit: Yi
Simulated: Dotter
Fit: Yi
Simulated: Yi
Fit: Dotter
95% Posterior Intervals (MSun)
logAge
8.4
8.6
8.8
9.0
9.1
9.2
9.4
9.6
True Initial Mass (MSun)
True Final Mass (MSun)

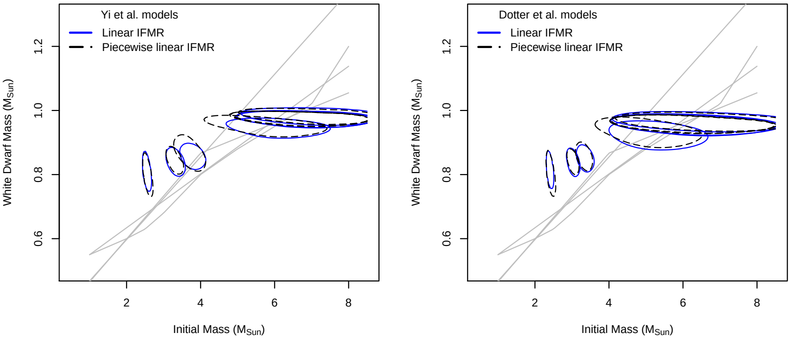
Data Analysis

- Analyzed three clusters: NGC 2477, the Hyades, and M35
- Results for NGC 2477 and the Hyades under both the Yi et al. (2001) and Dotter et al. (2008) models
- Results for M35 under the Yi et al. (2001) models (M35 is too young for the Dotter et al. (2008) models)

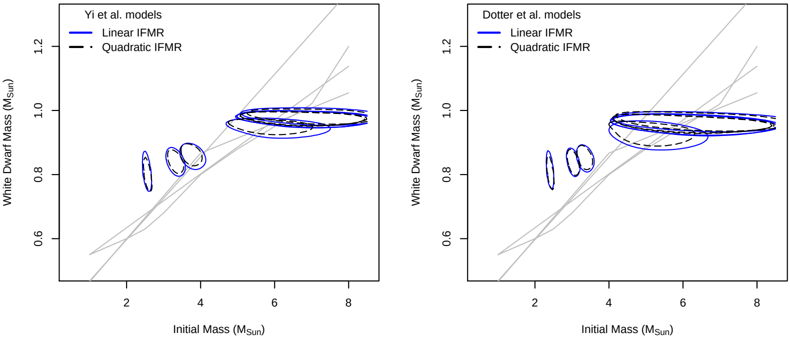
Initial and Final Mass Inferences



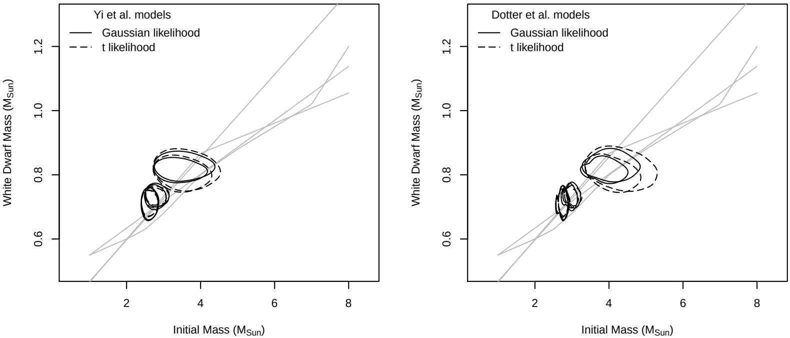
Sensitivity of NGC 2477 Inferences to IFMR Model



Sensitivity of NGC 2477 Inferences to IFMR Model



Sensitivity of Hyades Inferences to Error Model



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